

Anticrack model for pressure solution surfaces

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ABSTRACT

We propose that discrete solution surfaces originate at stress concentrations and propagate through rock as *anticracks*. As material is dissolved and removed, the anticrack walls move toward each other; stress and displacement fields are identical to those for the conventional opening crack, but with a change of sign. Observations of entire traces of solution surfaces are consistent with the anticrack concept: (1) the surfaces are bounded in extent; (2) the dissolved thickness varies from a maximum near the center to zero at the tips; and (3) the maximum dissolved thickness is proportional to the length of the surface. Local dissolution and in-plane propagation are suggested by the large isotropic compressive stress at the anticrack tip. Propagating solution surfaces will interact to form a regular array corresponding to some bulk strain rate. Anticracks may also interact with opening and shear cracks; observations of interacting solution surfaces, veins, and faults illustrate these configurations. Intersecting arrays of cracks, anticracks, and shear cracks operate to yield a mode of bulk deformation similar to diffusion-accommodated grain-boundary sliding in polycrystalline solids.

INTRODUCTION

Pressure solution is an important process in deformation and compaction of porous sedimentary rocks; in some cases it results in strains as great as 50% (Alvarez and others, 1978). Two contrasting modes of pressure solution are observed in limestones and sandstones. One mode is characterized by discrete *surfaces of solution*, which frequently show stylolite structures. The major amount of shortening is due to dissolution on these surfaces. This mode occurs both in tectonically deformed rocks where the solution surfaces comprise a spaced solution cleavage (Nickelsen, 1972; Groshong, 1975; Alvarez and others, 1978) and also in vertically compacted rock where the solution surfaces are bedding-parallel stylolite surfaces (Stockdale, 1922; Heald, 1955). The second mode is *pervasive pressure solution*, characterized by dissolution at the grain scale that is distributed throughout the rock. Dissolution takes place along grain contacts that lie normal to the direction of shortening, and the soluble components may be precipitated locally either into rock porosity (Heald, 1956), or as fibrous growths between grains that separate due to extension (Elliott, 1973).

We focus attention on the first mode of pressure solution and the discrete solution surfaces. Several outstanding problems are the following: How is an array of such surfaces established? How is the rock rheology to be described? Under what conditions does pervasive dissolution give way to that involving discrete sur-

faces? We propose a mechanical model that addresses these problems and that exhibits properties in agreement with observation.

No quantitative theory for deformation involving discrete solution surfaces has been developed, but work on this topic has been dominated by three concepts: (1) the location of solution surfaces is determined by pre-existing structures (Stockdale, 1922), notably bedding-parallel surfaces or joints; (2) the presence of clay enhances pressure solution; and (3) chemical or transport problems, especially the mechanism for solution, the modes of material transport away from solution surfaces, and the location of the sink, are central to or sufficient for the understanding of the process (Weyl, 1959; Robin, 1978).

The concept that the existence and location of solution surfaces are determined by pre-existing structure was clearly set forth by Stockdale (1922). He was of the opinion that "Stylolites originate . . . along a bedding plane, lamination plane, or crevice, where the circulation of ground waters, charged with carbon dioxide, is most free." Thus, the existence and distribution of discrete solution surfaces are set aside as points not needing further explanation, and it is implied that the fundamental property of a surface is its ability to act as an effective pathway for material transport. Heald (1955) concurred with Stockdale, but later he developed the concept that original clay partings could have served as loci for stylolite solution. Several mechanisms for this have been proposed (Weyl, 1959; Thomson, 1959; Sibley and Blatt, 1976). Sansone (1979) reported solution associated with tectonic shortening on two sets of surfaces on which the stylolite teeth lie in the shortening direction, at an acute

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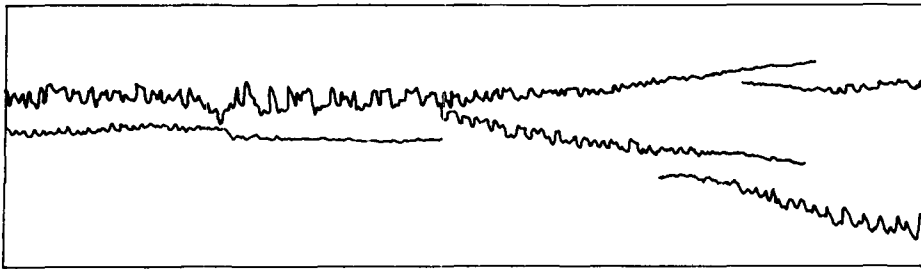


Figure 1. Terminations of solution surfaces (Stockdale, 1922, Fig. 16). Stylolite amplitude, about equal to thickness of rock dissolved, decreases gradually to zero at terminations.

angle to these surfaces. The solution surfaces are interpreted to have formed on pre-existing joints. Barrett (1964) gave a detailed description of spaced solution surfaces that developed in the compaction of a detrital limestone. Some of these surfaces did not form along pre-existing clay seams, because locally the surfaces transect primary stratification at a small angle. The pattern and morphology of the solution surfaces studied by Barrett are very similar to the spaced solution cleavage formed in tectonic shortening (Alvarez and others, 1978, Fig. 1). These authors suggested that increasing cleavage intensity involves the formation of new cleavage surfaces, not simply the reduction of spacing between original surfaces. It is clear that the distribution of solution surfaces in a rock body is not determined solely by pre-existing joint surfaces or clay seams.

SOLUTION SURFACES MODELED AS ANTICRACKS

To proceed further, we use several observations made by Stockdale (1922) of bedding-parallel solution surfaces in limestone. These observations are made of the entire traces of solution surfaces. Observations by other investigators have generally been restricted to outcrops, thin sections and drill cores with dimensions much smaller than those of the surfaces. Stockdale observed the following: (1) Solution surfaces are bounded in extent. (Solution surface terminations observed by Stockdale are shown in Fig. 1.) (2) The thickness of rock dissolved is roughly equal to the maximum height of the stylolite columns. This permits the thickness of rock dissolved along a seam to be estimated. (3) The dissolved thickness varies from a maximum in the central part of the surface to zero at its periphery. (4) The dissolved thickness at the center is proportional to the length or dimension of the surface. We consider the stylolite structures themselves to be "minor structures" that have no significant effect on the mechanics of the solution surfaces. Many solution surfaces do not show them, and their presence or absence should not affect the inferences drawn below.

The basic implication of the bounded extent of solution surfaces is that the displacement must die off toward the terminations and must be accommodated by deformation concentrated near the termination. For typical mechanical behavior, this concentrated deformation will lead to a stress concentration that suggests that propagation of the surface may occur; the example of a crack loaded in tension is familiar. This, in turn, suggests that solution surfaces originate at flaws or heterogeneities in the rock where stress concentrations occur. Durney (1974) anticipated the basic features of this concept insofar as he treated a solution surface as a thin crack, normal to a uniaxial compressive stress, that propagates in its plane due to stress concentrations at its tips.

Important distinctions between this process and solution along pre-existing surfaces or seams should be mentioned. Excluding the flaws or heterogeneities that may be expected in

natural materials, the original rock may be essentially uniform. Also, the orientation, spacing, and magnitude of the structures produced will depend strongly on the imposed deformation or on the rate of deformation. Pre-existing structures and lithological heterogeneities and layering certainly affect the development of solution surfaces, but these are not necessary. Moreover, the way in which solution surfaces are established in the presence of such features is not explained by the earlier concepts.

For an example, we consider a homogeneous, isotropic, linearly elastic medium. If the grain structure does not undergo compaction or pervasive grain-scale pressure solution, we expect the removal of rock by dissolution on a discrete surface to produce an elastic response in the surrounding rock. If pervasive pressure solution takes place only near the termination of the solution surface, it is still appropriate to consider an elastic response of the medium. An analogous situation exists when plastic deformation takes place at the tip of a crack loaded in tension (Lawn and Wilshaw, 1975). For simplicity, the elastic problem considered is two-dimensional, and the cut that represents the solution surface has infinite (very large) depth normal to the cross section shown in Figure 2. This special geometry may be modified to a more realistic one without changing the results qualitatively. Notation and sign conventions, for stress and displacements, and the coordinate axes are illustrated in Figure 2.

In a thought experiment designed to mimic the deformation near a solution surface, we imagine that a thin lamina of material is cut from the elastic body in its unloaded state to make a crack. Then the walls of the cavity are pressed together in perfect contact to form the solution surface by an applied load with maximum compressive stress normal to the surface. We might choose the variation in lamina thickness, to correspond to the distribution of thickness of rock dissolved along a solution surface, as measured in the field. In the absence of such measurements, we consider that the thickness is appropriate to produce a uniform compressive stress, $\sigma_{yy}^C = -\sigma^C$, and no shear stress, $\sigma_{xy}^C = 0$, on the surface. Uniform remote compressive stresses $\sigma_{yy} = -\sigma^\infty$ and $\sigma_{xx} = -k\sigma^\infty$ act normal and parallel to the surface. Anticipating later results concerning the propagation of solution surfaces, we choose $\sigma_{xy}^\infty = 0$, so that the surface coincides with a principal stress plane. If $\sigma^C = \sigma^\infty$, no relative displacements of the opposing sides of the surface take place, and the stress throughout the body is uniform. The case of interest is when $\sigma^C > \sigma^\infty$, corresponding to a relaxation of the compressive stress acting on the surface. The solution to this problem is the same as that for the opening of a crack, $\sigma^C < \sigma^\infty$, except that the signs of the displacements and stresses have changed. Thus, we refer to this as the *anticrack* solution.

Displacements of the anticrack walls formally correspond to an interpenetration of opposing sides, a state that would appear to lack physical realism. Indeed, for mechanicians interested in certain crack solutions, interpenetration has presented a prob-

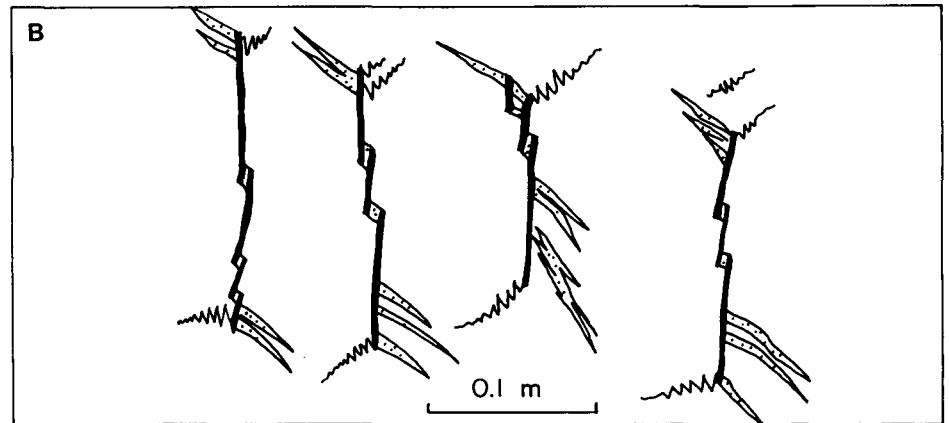
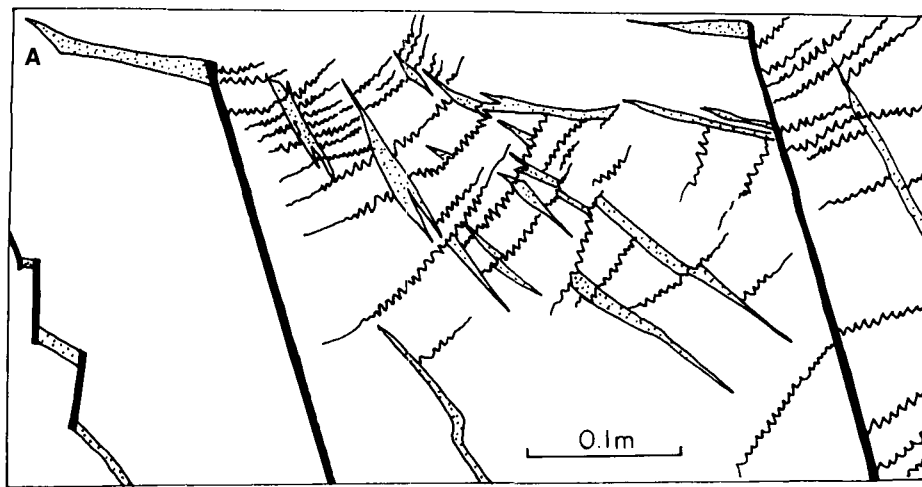


Figure 3. Examples of solution surfaces interacting with other structures (Rispoli, 1981). A. Array of orthogonal solution surfaces and veins. B. Faults, solution surfaces, and veins.

originated by the simultaneous propagation of anticracks and cracks. However, the analysis of a single, elementary structure provides sufficient insight. Consider the stress field in the vicinity of an opening crack (Fig. 2) subject to $\sigma^C < \sigma^\infty$ and no shear loads. The normal stress distribution across an orthogonal plane passing through the center of the crack is derived following Eftis and Liebowitz (1972). The deviation from the applied value is symmetrically distributed about the crack plane (Fig. 4A). This stress has a minimum value (maximum compression) at the crack surface, increases to zero at about $0.8a$, takes a broad maximum value and then tends asymptotically to zero. The large compression at the crack surface indicates that anticrack growth will initiate there in response to crack opening. Because the stress trajectories emanating from this point are normal to the crack, we expect propagation of the anticrack outward in both directions along the y axis (Fig. 4B). The sign change beyond $0.8a$ suggests a stabilizing effect for an anticrack that might approach the length of the crack. Thus, we expect the propagating anticrack not to greatly exceed the length of the crack.

For an anticrack under load $\sigma^C > \sigma^\infty$, the normal stress distribution is identical to that in Figure 4A, but of opposite sign. Thus, under certain loading conditions, the crack-anticrack orthogonal pairs tend to couple together as a "unit cell" of deformation, each enhancing but limiting the propagation of the

other. If the crack-anticrack pair represented a closed system with no removal or addition of the soluble rock component, one would set the crack volume equal to the anticrack "anti-volume" and solve for the equilibrium crack and anticrack lengths for a particular loading. Because the two sets of structures are laid over each other, the mechanical interactions include those between structures of the same kind and between cracks and anticracks.

Coupling of Solution Surfaces, Veins, and Faults

Typically, rock masses are cut by many weak surfaces that can slide if subjected to a sufficient shear stress. Rispoli (1981) has mapped structures of this type in limestones (Fig. 3B is a reproduction of one of those maps). The following observations bear emphasizing: (1) solution surfaces and veins emanate from near the fault tips; (2) displacements on these surfaces decrease markedly away from the fault over distances less than the fault length; and (3) the configuration of solution surfaces and veins is antisymmetric with respect to the fault. The elastic stress field due to interaction of a shear crack with opening cracks and anticracks provides a rationalization of these observations.

We seek insight by considering the stress field about a single surface (Fig. 2) subject to uniform remote stress $\sigma_{xx}^\infty = \sigma_{yy}^\infty = 0$, $\sigma_{xy}^\infty = \sigma^\infty$, and a uniform surface stress $\sigma_{yy}^C = 0$,

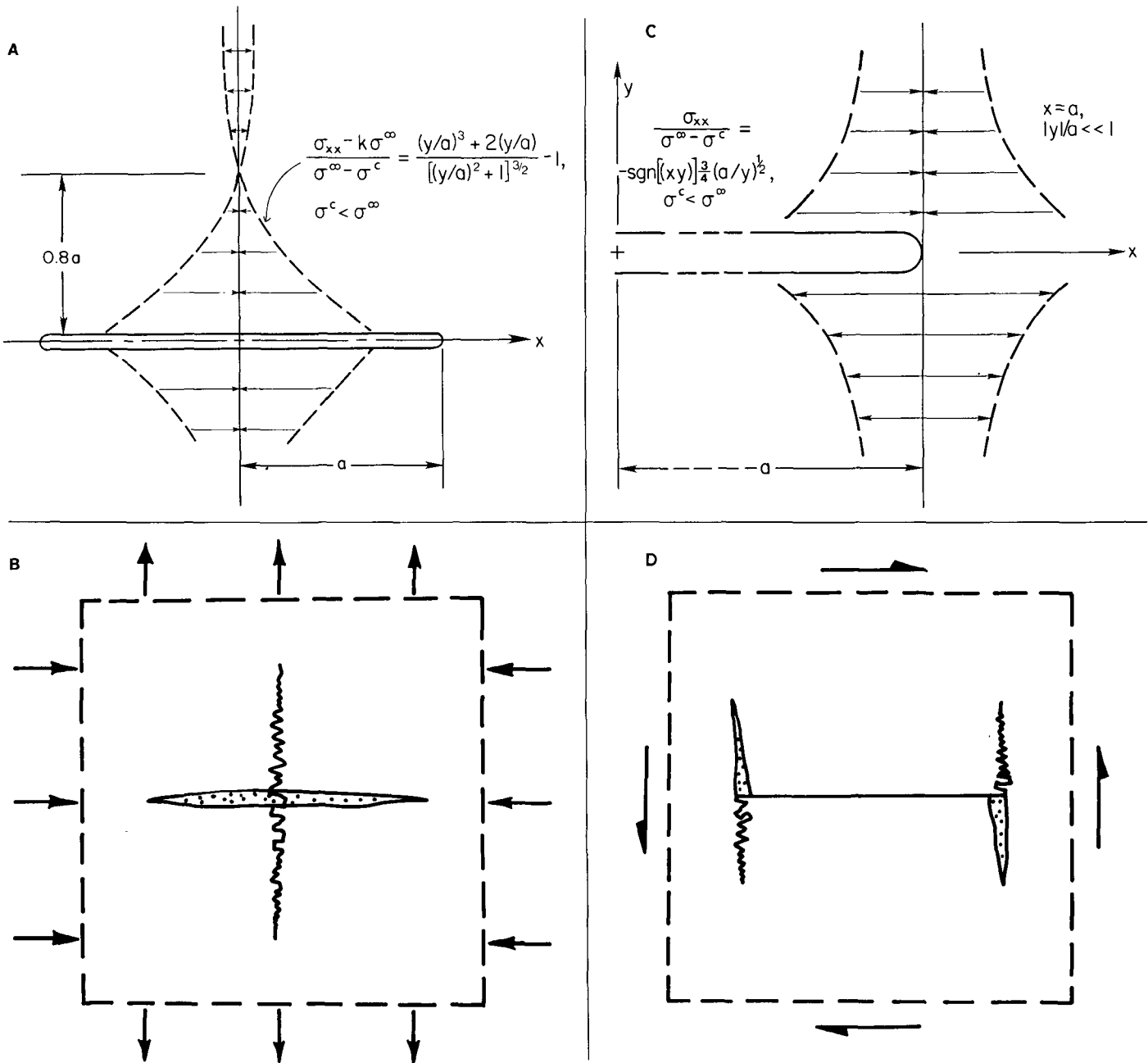


Figure 4. Stress analysis indicating manner in which opening cracks and shear cracks might interact with anticracks. A. Induced compressive stress near center of opening crack. B. Orthogonal arrangement of crack-anticrack couple in biaxial stress field. C. Induced compression and tension on opposite sides of shear-crack tip. D. Arrangement of shear crack, crack, and anticrack in shear stress field.

$\sigma_{xy}^c = \sigma^c$. This shear loading produces no slip on the shear crack if $\sigma^\infty = \sigma^c$, right-lateral slip if $\sigma^\infty > \sigma^c$, and left-lateral slip if $\sigma^\infty < \sigma^c$. Slip induces stresses that are concentrated near the tips of the shear crack. The normal stress distribution across an orthogonal plane passing through the tip (Lawn and Wilshaw, 1975) is antisymmetric about the coordinate axes (Fig. 4C). This suggests the configuration of cracks and anticracks shown in Figure 4D. The stress is singular at the shear crack tips and decays as $1/\sqrt{y}$. Thus, we expect the cracks and anticracks to taper away from the shear crack and, if driven only by the stress concentration near the shear-crack tip, to remain short relative to the shear crack. The solution surfaces and veins in Figure 3B are not orthogonal to the faults but extend outward at a steep angle. This angularity is consistent with the direction of stress trajectories emanating from the tip region of a shear crack.

Under certain loading conditions, the three surfaces arranged

as in Figure 4D couple together as a "unit cell" of deformation, each enhancing the driving stress on the others. The cracks form convenient sinks for material removed from the anticrack surfaces. For a given loading and shear-crack length, the condition of no removal or addition of soluble constituents provides a constraint on the lengths of cracks and anticracks. A single unit cell may interact with others nearby, to complicate the stress field, as in Rispoli's example (Fig. 3B), but the essence of the deformation is contained in the single structure.

ARRAYS OF SOLUTION SURFACES, VEINS, AND FAULTS AND BULK DEFORMATION OF A ROCK MASS

The simple examples discussed above suggest how an intersecting array of solution surfaces, veins, and faults might be established in a rock mass by propagation and interaction. These three kinds of structures provide a complete set of mechanisms

for volume-conserving bulk deformation of the rock mass in which little inelastic deformation need be taken up within the blocks bounded by the structures. This is analogous to the way in which subduction zones, spreading ridges, and transform faults provide for an area-conserving bulk deformation of Earth's surface in plate tectonics.

If the deformation rock has a large initial porosity (for example, a compacting limestone), bulk shortening may take place, with local conservation of the solid rock mass, but not its volume, as pore fluids are expelled. To effect such a deformation, only solution surfaces are developed. If stresses are sufficient to cause faulting or if pre-existing weak surfaces are present, all three types of structures may be involved in the bulk deformation. The mode of bulk deformation in this case would be closely equivalent to diffusion-accommodated grain-boundary sliding in polycrystalline materials (Raj and Ashby, 1971; Elliott, 1973), except that the "grain size" (block size) would be established in the course of deformation.

An orthogonal array of solution surfaces and veins can provide for the volume-conserving deformation of a tight rock. Consider a regular array of such structures with common spacing d . Let x denote the direction normal to the solution surfaces. The rate of shortening may be evaluated by methods analogous to those in Raj and Ashby (1971) or in Weyl (1959) by supposing that the dissolved material is transported to sites of deposition by diffusion in a fluid film present along the solution surfaces and vein centers. The resulting strain rate is

$$\dot{\epsilon}_{xx} = 6(c_0/RT)V_0^2(D\delta/d^3)(\sigma_{xx} - \sigma_{yy}), \quad (3)$$

where $\sigma_{xx} - \sigma_{yy}$ is the difference of the applied components of stress, c_0 is the mean concentration of the solid component in the fluid film, V_0 is the specific volume of the solid, D is the intrinsic diffusivity of the solute in water, δ is the effective thickness of the film, R is the gas constant, and T is the absolute temperature.

The strong dependence of strain rate on the block size or surface spacing d is noteworthy. If the strain rate is externally imposed, the stress will be determined by the block size. This, however, cannot exceed the tensile yield stress of the rock, a few tens of bars, since new veins, and presumably solution surfaces as well, would then form, reducing the block size. Evaluating the parameters in equation 3 for limestone at a temperature of 100 °C ($c_0 = 3 \times 10^{-7}$ M/cm³, $V_0 = 36.8$ cm³/M, $d \cong 10^{-5}$ cm²/s, $\delta \cong 10^{-3}$ cm), we obtain $\dot{\epsilon}_{xx} = 10^{-14}(\sigma_{xx} - \sigma_{yy})/d^3$ s⁻¹, where the stress difference is to be expressed in bars and d is to be expressed in centimetres. Taking $\sigma_{xx} - \sigma_{yy} = 30$ bar, d will range from about 10 cm to 1 cm as the strain rate ranges from 10⁻¹⁵ s⁻¹ to 10⁻¹⁴ s⁻¹. These values of the block size are consistent with those observed in limestones tectonically deformed by this process, including the examples shown in Figure 3.

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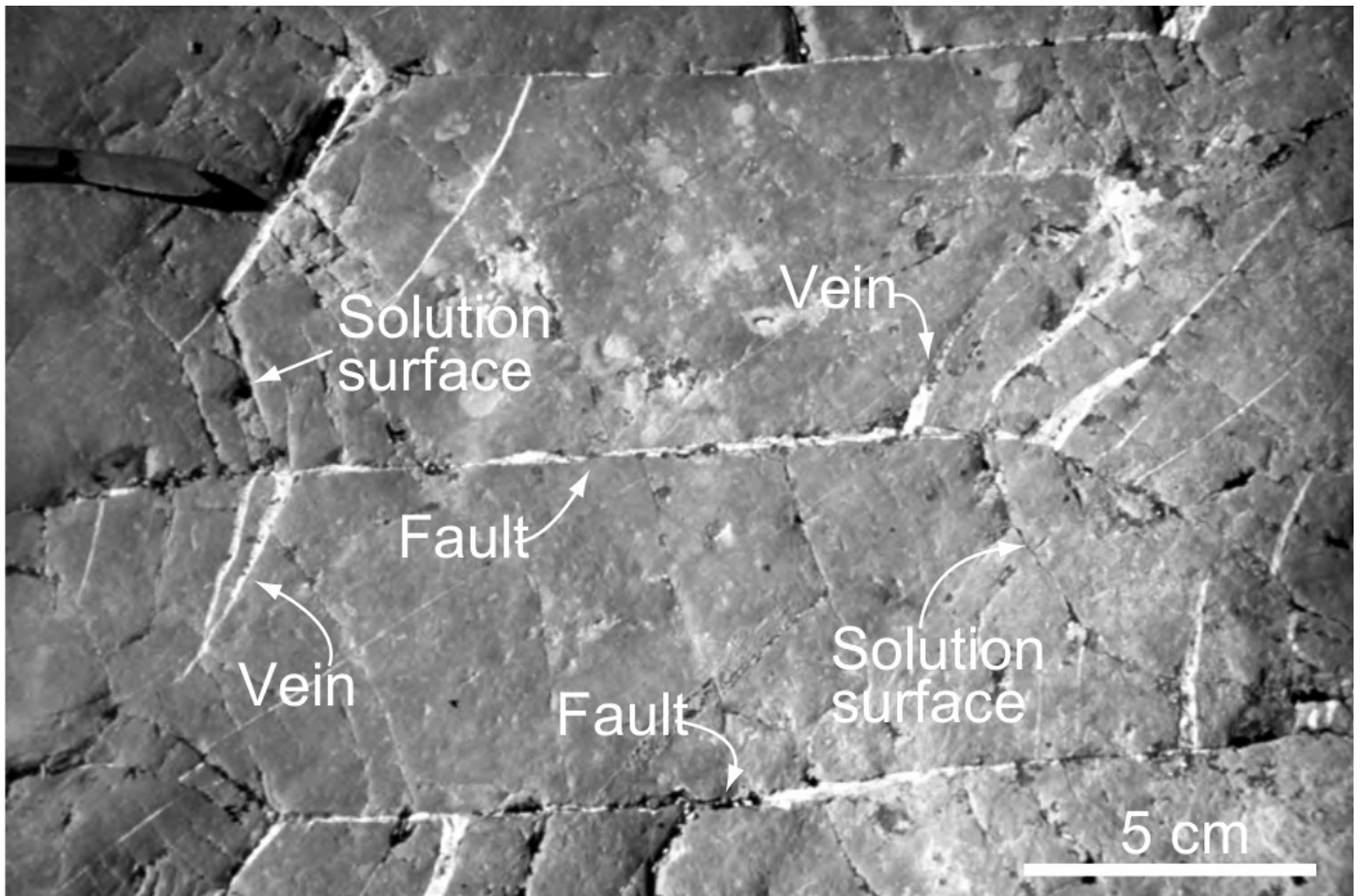
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Outcrop photograph from Les Matelles in southern France showing three small faults with traces parallel to the top of the photograph. Each fault has an antisymmetric distribution of veins (filled with white calcite) and solution surfaces (dark wavy bands).

Photograph by J.-P. Petit, from Petit and Mattauer (1995), "Palaeostress superimposition deduced from mesoscale structures in limestone: the Matelles exposure, Languedoc, France", *Journal of Structural Geology* 17(2): 245–56.
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