Elementary engineering fracture mechanics

DAVID BROEK

1982
MARTINUS NIJHOFF PUBLISHERS
THE HAGUE / BOSTON / LONDON

1 Summary of basic problems and concepts

1.1 Introduction

Through the ages the application of materials in engineering design has posed difficult problems to mankind. In the Stone Age the problems were mainly in the shaping of the material. In the early days of the Bronze Age and the Iron Age the difficulties were both in production and shaping. For many centuries metal-working was laborious and extremely costly. Estimates go that the equipment of a knight and horse in the thirteenth century was of the equivalent price of a Centurion tank in World War II.

With the improving skill of metal working, applications of metals in structures increased progressively. Then it was experienced that structures built of these materials did not always behave satisfactorily, and unexpected failures often occurred. Detailed descriptions of castings and forgings produced in the Middle Ages exist. When judged with present day knowledge, these production methods must have been liable to build important technical deficiences into the structure. This must have made gunners pray—when igniting the charge—that the projectile would be properly delivered and the barrel not blown up ...

The vastly increasing use of metals in the nineteenth century caused the number of accidents and casualties to reach unknown levels. The number of people killed in railway accidents in Great Britain was in the order of two hundred per year during the decade 1860–1870. Most of the accidents were a result of derailing caused by fractures of wheels, axles or rails. Anderson [1] has recently made an interesting compilation of accident reports from the last two hundred years. A few quotations follow:

"On the 19th of March 1830 about 700 persons assembled on the Montrose suspension bridge to witness a boat race, when one of the main chains gave way... and caused considerable loss of life."

"On the 22nd of January 1866, a portion of the roof of the Manchester railway station fell, causing deaths of two men. The accident was caused by failure of cast-iron struts connected..."

"The failure of a large gas tank in New York occurred on December 13, 1898, killing and injuring a number of people and destroying considerable surrounding property."

"A high pressure water main burst at Boston, January 3, 1913 and flooded the district..."

"Engineering, February 1866. With some *fifty to sixty* boiler explosions annually in the United Kingdom attended as they are with loss of many lives and destruction of property, is it not time that the Goverment should appoint a commission to inquire into the subject?"

"The most serious railroad accident of the *week* occurred April 20 (1887) and was caused by the breaking of a drawbar. Three were killed and two fatally injured."

"The most serious railroad accident of the *week* occurred May 27 (1887). The bursting of a wheel caused the deaths of six people."

"The most serious railroad accident of the *week* occurred June 23 (1887) and was caused by a broken rail. One man was killed."

"The most serious railroad accident of the *week* occurred on July 2 (1887) and was caused by the breaking of an axle."

Some of these accidents were certainly due to a poor design, but it was gradually discovered that material deficiencies in the form of pre-existing flaws could initiate cracks and fractures. Prevention of such flaws would improve structural performance. Better production methods together with increasing knowledge and comprehension of material properties led to a stage where the number of failures was reduced to more acceptable levels.

A new era of accident prone structures started with the introduction of all-welded designs. Out of 2500 Liberty ships built during World War II, 145 broke in two and almost 700 experienced serious failures. The same disaster struck many bridges and other structures. Information on these failures was also given by Anderson [1] and more specifically e.g. by Biggs [2].

The failures often occurred under conditions of low stresses (several ships failed suddenly while in the harbour) which made them seemingly inexplicable. As a result extensive investigations were initiated in many countries and especially in the United States of America. This work revealed that here again, flaws and stress concentrations (and to a certain extent internal stresses) were responsible for failure.

The fractures were truly brittle: they were accompanied by very little plastic deformation. It turned out that the brittle fracture of steel was promoted by low temperatures and by conditions of triaxial stress such as may exist at a sharp notch or a flaw. Under these circumstances structural steel can fracture by cleavage (chapter 2) without noticeable plastic deformation. Above a certain temperature, called the transition temperature, the steel behaves in a ductile manner. The transition temperature may go up as a result of the heat cycle during the welding process.

At present, brittle fractures of welded structures built out of low strength structural steels can be satisfactorily prevented. It has to be ensured that the material is produced to have a low transition temperature and that the welding process does not raise the ductile-brittle transition. Large stress concentrations should be avoided and the welds should be checked to be virtually free of defects.

After World War II the use of high strength materials has increased considerably. These materials are often selected to realize weight savings. Simultaneously, stress analysis methods were developed which enable a more reliable determination of local stresses. This permitted safety factors to be reduced resulting in further weight savings. Consequently, structures designed in high strength materials have only low margins of safety. This means that service stresses (sometimes with the aid of an aggressive environment) may be high enough to induce cracks, particularly if pre-existing flaws or high stress concentrations are present. The high strength materials have a low crack resistance (fracture toughness): the residual strength under the presence of cracks is low. When only small cracks exist, structures designed in high strength materials may fail at stresses below the highest service stress they were designed for.

Low stress fractures induced by small cracks are, in many aspects, very similar to the brittle fractures of welded low-strength steel structures. Very little plastic deformation is involved; the fracture is brittle in an engineering sense, although the micromechanism of separation is the same as in ductile fracture. The occurrence of low stress fracture in high strength materials induced the development of *Fracture Mechanics*.

Engineering fracture mechanics can deliver the methodology to compensate the inadequacies of conventional design concepts. The conventional design criteria are based on tensile strength, yield strength and buckling stress. These criteria are adequate for many engineering structures, but they are insufficient when there is the likelihood of cracks. Now, after approximately two decades of development, fracture mechanics have become a useful tool in design with high strength materials.

This first chapter is an introduction to fracture mechanics. Section 1.2 presents a survey of the problems that can be solved with fracture mechanics concepts; it gives an outline of the total field of fracture mechanics, which is much broader than is often thought. The rest of the chapter is a brief introductory summary of the concepts of fracture mechanics. All these subjects receive ample attention in later chapters.

1.2 A crack in a structure

Consider a structure in which a crack develops. Due to the application of repeated loads or due to a combination of loads and environmental attack this crack will grow with time. The longer the crack, the higher the stress concentration induced by it. This implies that the rate of crack propagation will increase with time. The crack propagation as a function of time can be represented by a rising curve as in figure 1.1a. Due to the presence of the crack the strength of the structure is decreased: it is

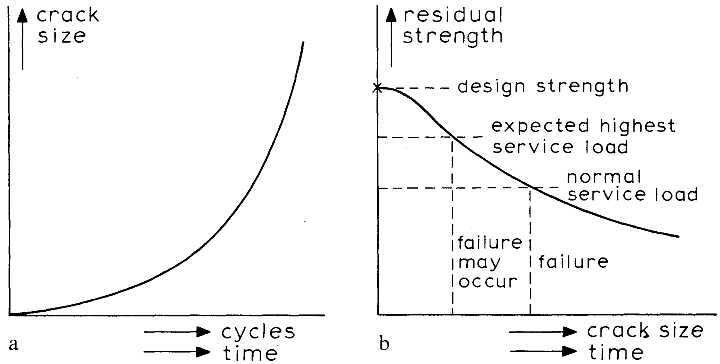

Fig. 1.1. The engineering problem
a. Crack growth curve; b. Residual strength curve

lower than the original strength it was designed for. The residual strength of the structure decreases progressively with increasing crack size, as is shown diagrammatically in figure 1.1b. After a certain time the residual strength has become so low that the structure cannot withstand accidental high loads that may occur in service. From this moment on the structure is liable to fail. If such accidental high loads do not occur, the crack will continue to grow until the residual strength has become so low that fracture occurs under normal service loading. Many structures are designed to carry service loads that are high enough to initiate cracks, particularly when pre-existing flaws or stress concentrations are present. The designer has to anticipate this possibility of cracking and consequently he has to accept a certain risk that the structure will fail. This implies that the structure can have only a limited lifetime. Of course, the probability of failure should be at an acceptable low level during the whole service life. In order to ensure this safety it has to be predicted how fast cracks will grow and how fast the residual strength will decrease: Making these predictions and developing prediction methods are the objects of fracture mechanics.

With respect to figure 1.1 fracture mechanics should be able to answer the following questions:

a. What is the residual strength as a function of crack size?
b. What size of crack can be tolerated at the expected service load; i.e. what is the critical crack size?
c. How long does it take for a crack to grow from a certain initial size to the critical size?
d. What size of pre-existing flaw can be permitted at the moment the structure starts its service life?
e. How often should the structure be inspected for cracks?

Fracture mechanics provide satisfactory answers to some of these questions and useful answers to the others. As depicted in figure 1.2 several disciplines are involved in the development of fracture mechanics design procedures. At the right end of the scale is the engineering load-and-stress analysis. Applied mechanics provide the crack tip stress fields as well as the elastic and (to a certain extent) plastic deformations of the material in the vicinity of the crack. The predictions made about fracture strength can be checked experimentally. Material Science concerns itself with the fracture processes on the scale of atoms and dislocations to that of impurities and grains. From a comprehension of these processes the

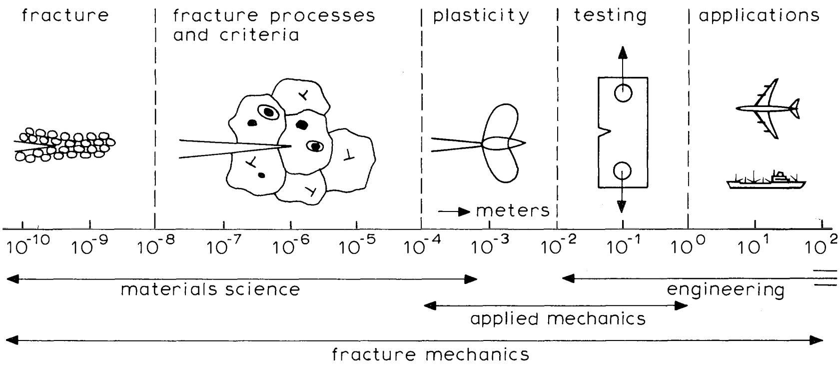

Figure 1.2. The broad field of fracture mechanics

criteria which govern growth and fracture should be obtainable. These criteria have to be used to predict the behaviour of a crack in a given stress-strain field. An understanding of fracture processes can also provide the material parameters of importance to crack resistance; these have to be known if materials with better crack resistance are to be developed.

In order to make a successful use of fracture mechanics in engineering application it is essential to have some knowledge of the total field of figure 1.2. This book attempts to provide a basic understanding of this field.

1.3 The stress at a crack tip

A crack in a solid can be stressed in three different modes, as illustrated in figure 1.3. Normal stresses give rise to the "opening mode" denoted as

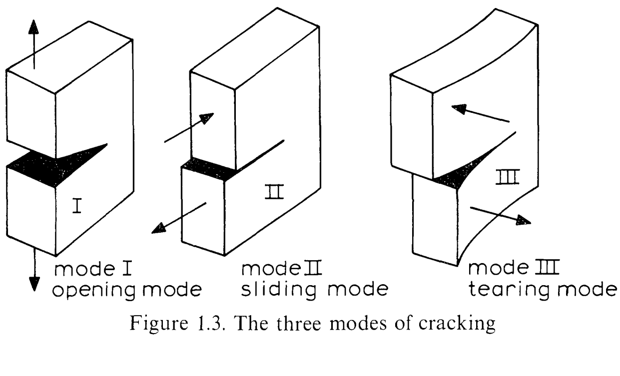

Figure 1.3. The three modes of cracking

mode I. The displacements of the crack surfaces are perpendicular to the plane of the crack. In-plane shear results in mode II or "sliding mode": the displacement of the crack surfaces is in the plane of the crack and perpendicular to the leading edge of the crack. The "tearing mode" or mode III is caused by out-of-plane shear. Crack surface displacements are in the plane of the crack and parallel to the leading edge of the crack. The superposition of the three modes describes the general case of cracking. Mode I is technically the most important; the discussions in this introductory chapter are limited to mode I.

Consider a through-the-thickness mode I crack of length $2a$ in an infinite plate, as in figure 1.4. The plate is subjected to a tensile stress σ at infinity. As discussed in chapters 3 and 13 there are several ways to calculate the elastic stress field at the crack tip. An element $\mathrm{d}x\mathrm{d}y$ of the plate at a distance r from the crack tip and at an angle θ with respect to the crack plane, experiences normal stresses σ_x and σ_y in X and Y directions and a shear stress τ_{xy}. These stresses can be shown [3–6] to be (see chapter 3):

$$
\begin{aligned}
\sigma_x &= \sigma\sqrt{\frac{a}{2r}}\cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]\\
\sigma_y &= \sigma\sqrt{\frac{a}{2r}}\cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]\\
\tau_{xy} &= \sigma\sqrt{\frac{a}{2r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\\
\sigma_z &= 0 \text{ (plane stress)}\\
\sigma_z &= \nu(\sigma_x+\sigma_y) \text{ (plane strain)}\,. \qquad (1.1)
\end{aligned}
$$

(Note that a is the semi-crack length).

As should be expected, in the elastic case the stresses are proportional to the external stress σ. They vary with the square root of the crack size and they tend to infinity at the crack tip where r is small. The distribution of the stress σ_y as a function of r at $\theta=0$ is illustrated in figure 1.5. For large r the stress σ_y approaches zero, while it should go to σ. Apparently, eqs (1.1) are valid only for a limited area around the crack tip. Each of the equations represents the first term of a series. In the vicinity of the crack tip these first terms give a sufficiently accurate description of the crack tip stress fields, since the following terms are

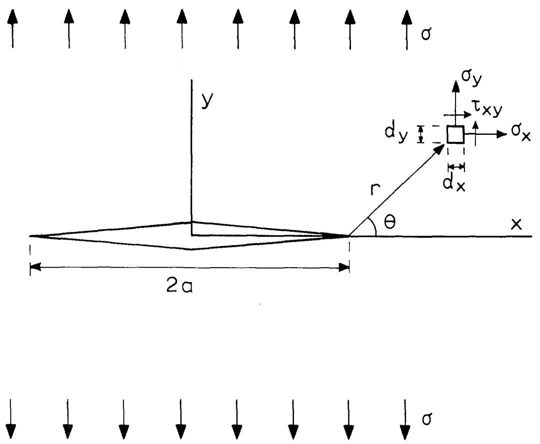

Figure 1.4. Crack in an infinite plate

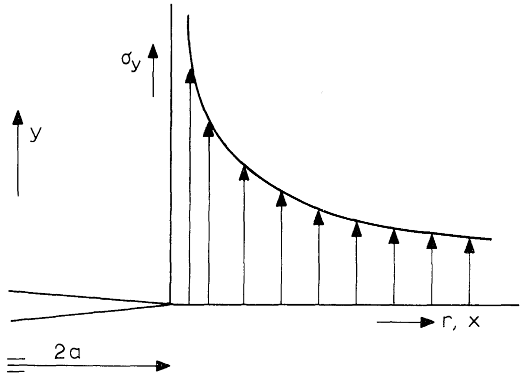

Figure 1.5. Elastic stress σ_y at the crack tip

small compared to the first. Further away, more terms will have to be taken into account (chapter 3).

The functions of the coordinates r and θ in eqs (1.1) are explicit. The equation can be written in the generalized form

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) \text{ with } K_I = \sigma\sqrt{\pi a}\,. \tag{1.2}$$

The factor K_I is known as the "stress intensity factor"*, where the subscript I stands for mode I. The whole stress field at the crack tip is known when the stress intensity factor is known. Two cracks, one of size $4a$ the other of size a have the same stress field at their tips if the first crack is loaded to σ and the other to 2σ. In that event K_I is the same for both cracks.

Eq (1.2) is an elastic solution, which does not prohibit that the stresses become infinite at the crack tip. In reality this cannot occur: plastic deformation taking place at the crack tip keeps the stresses finite. An impression of the size of the crack tip plastic zone can be obtained [7, 8] by determining to which distance r_p^* from the crack tip the elastic

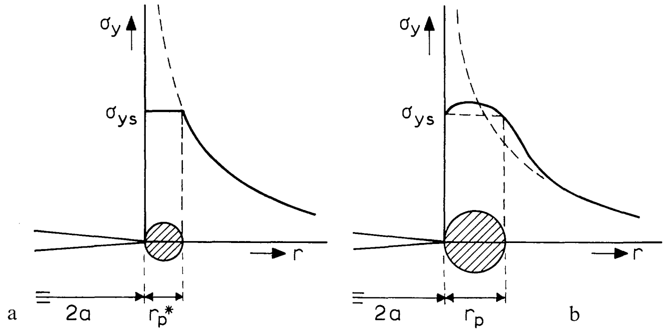

Figure 1.6. Plastic zone at crack tip
a. Assumed stress distribution; b. Approximate stress distribution

stress σ_y is larger than the yield stress σ_{ys} (fig. 1.6a). Substituting $\sigma_y = \sigma_{ys}$ into eq (1.1) for σ_y and taking the plane $\theta = 0$ it follows that:

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r_p^*}} = \sigma_{ys} \quad \text{or} \quad r_p^* = \frac{K_I^2}{2\pi\sigma_{ys}^2} = \frac{\sigma^2 a}{2\sigma_{ys}^2}. \tag{1.3}$$

In reality the plastic zone is somewhat larger (figure 1.6b). General expressions for the plastic zone size are discussed in chapter 4. It may suffice here to point out that r_p^* can be directly expressed as a function of the stress intensity factor and the yield stress.

In a foregoing paragraph it was stated that elastic cracks of different

* Note that K_I differs from the stress concentration factor, k_t, both in dimensions and meaning. The latter is the ratio between the maximum stress and the nominal stress in a notched sample.

sizes but with the same K_I have similar stress fields. The question arises whether this argument still holds if plastic deformation occurs. Cracks loaded to the same K_I have plastic zones of equal size according to eq (1.3). Outside the plastic zone the stress field will still be the same. If the two cracks have equal plastic zones and the same stresses acting at the boundary of the zone, then the stresses and strains within the plastic zone must be equal.

In other words the stress intensity factor is still likely to determine the stress field. It also determines what occurs inside the plastic zone. K is a measure for all stresses and strains. Crack extension will occur when the stresses and strains at the crack tip reach a critical value. This means that fracture must be expected to occur when K_I reaches a critical value K_{Ic}. The critical K_{Ic} may be expected to be a material parameter.

One can take a plate with a crack of known size and pull this plate to fracture in a tensile machine. From the fracture load the failure stress σ_c can be calculated. Then it follows that the critical value of the stress intensity factor at the moment of failure is given by:

$$K_{Ic} = \sigma_c \sqrt{\pi a} \,. \tag{1.4}$$

If K_{Ic} is a material parameter the same value should be found by testing another specimen of the same material but with a different size of the crack. Within certain limits this is indeed the case. On the basis of this K_{Ic} value the fracture strength of cracks of any size in the same material can be predicted. It can also be predicted which size of crack can be tolerated in the material if stressed to a given level.

In reality the situation is slightly more complicated. First of all, the used expression for the stress intensity factor is valid only for an infinite plate. For a plate of finite size the formula becomes (chapter 3):

$$K_I = \sigma \sqrt{\pi a} \, f\left(\frac{a}{W}\right) \tag{1.5}$$

where W is the plate width. The function $f(a/W)$ has to be known before K_{Ic} can be determined. Of course, $f(a/W)$ approaches unity for small values of a/W. Secondly, a restriction has to be made as to the transverse strains in the plate. A consistent K_{Ic} value can only be obtained from a test if the displacements in the thickness direction of the plate are sufficiently constrained, i.e. when there is a condition of plane strain. This occurs when the plate has a large enough thickness (chapters 4, 7). If deformations in the thickness direction can take place freely (plane stress situation) the critical stress intensity factor depends upon plate thickness (chapters 4, 8).

K_{Ic} is a measure for the crack resistance of a material. Therefore K_{Ic} is called the "plane strain fracture toughness". Materials with low fracture toughness can tolerate only small cracks. Typical values of the fracture toughness of three high strength materials are given in table 1.1.

TABLE 1.1

	tensile strength σ_u			yield strength σ_{ys}			fracture toughness K_{Ic}
	MN/m^2	kg/mm^2	ksi	MN/m^2	kg/mm^2	ksi	
4340 steel	1820	185	264	1470	150	214	46 MN/m$^{3/2}$ = 150 kg/mm$^{3/2}$ = 42 ksi$\sqrt{\text{in}}$
Maraging 300 steel	1850	188	268	1730	177	250	90 MN/m$^{3/2}$ = 290 kg/mm$^{3/2}$ = 82 ksi$\sqrt{\text{in}}$
7075-T6 Al. alloy	560	57	81	500	51	73	32 MN/m$^{3/2}$ = 104 kg/mm$^{3/2}$ = 30 ksi$\sqrt{\text{in}}$

Note the typical dimensions of fracture toughness. The conversion of units is:
1 MN/m$^{3/2}$ = 3.23 kg/mm$^{3/2}$ = 0.925 ksi$\sqrt{\text{in}}$
1 kg/mm$^{3/2}$ = 0.31 MN/m$^{3/2}$ = 0.287 ksi$\sqrt{\text{in}}$
1 ksi$\sqrt{\text{in}}$ = 1.081 MN/m$^{3/2}$ = 3.49 kg/mm$^{3/2}$.

The size of crack that can be tolerated in the materials of table 1.1. before the strength has decreased to half the original strength can be determined from:

$$\sigma_c = \frac{K_{Ic}}{\sqrt{\pi a}} = \frac{\sigma_u}{2} \quad \text{or} \quad a = \frac{4K_{Ic}^2}{\pi \sigma_u^2}. \tag{1.6}$$

One finds that a crack of $2a$=1.67 mm can be tolerated in the 4340 steel, whereas the maraging steel allows a crack of $2a$=6.06 mm and the aluminium alloy $2a$=8.48 mm.

In figure 1.7a the residual strength of the three materials is plotted as a function of crack length. These curves follow from $\sigma_c = K_{Ic}/\sqrt{\pi a}$. The

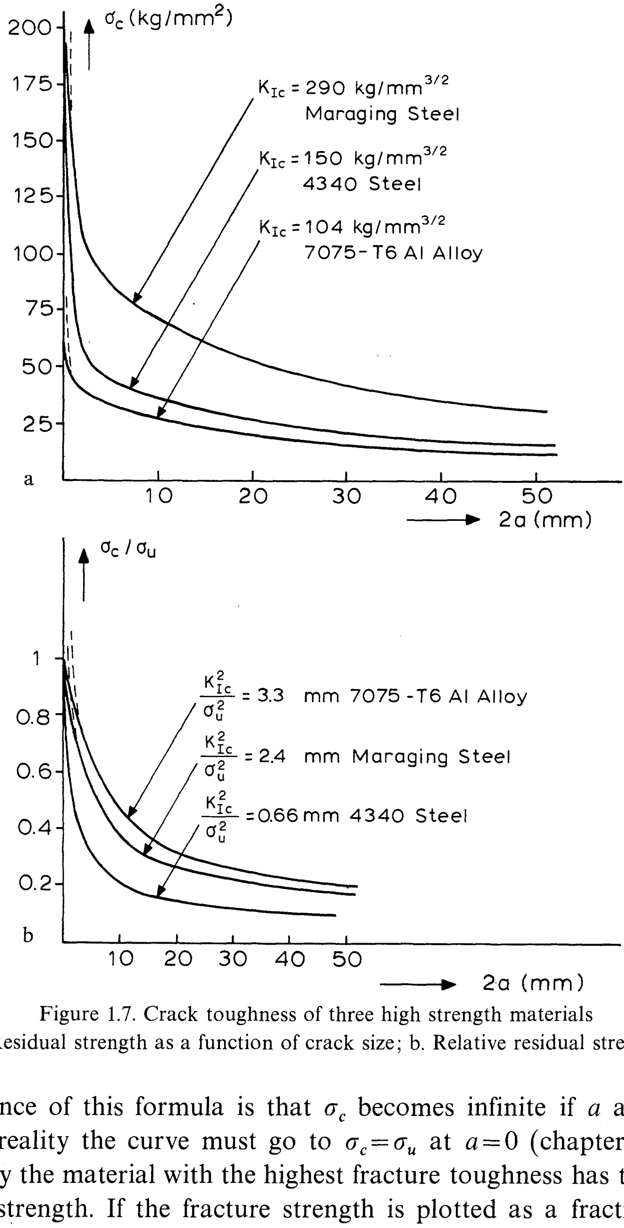

Figure 1.7. Crack toughness of three high strength materials
a. Residual strength as a function of crack size; b. Relative residual strength

consequence of this formula is that σ_c becomes infinite if a approaches zero. In reality the curve must go to $\sigma_c=\sigma_u$ at $a=0$ (chapters 7, 8, 9). Obviously the material with the highest fracture toughness has the highest residual strength. If the fracture strength is plotted as a fraction of the original (crack free) strength, σ_c/σ_u, the picture is completely different (figure 1.7b). The aluminium alloy tolerates longer cracks than the other materials for a percentage-wise equal loss in strength. This is due to the fact that the aluminium alloy has the highest ratio of toughness to tensile strength (indicated in figure 1.7b).

1.4 The Griffith criterion

Although fracture mechanics have been developed mainly in the last two decades, one of the basic equations was established already in 1921 by Griffith [9, 10]. Consider an infinite cracked plate of unit thickness with a central transverse crack of length $2a$. The plate is stressed to a stress σ

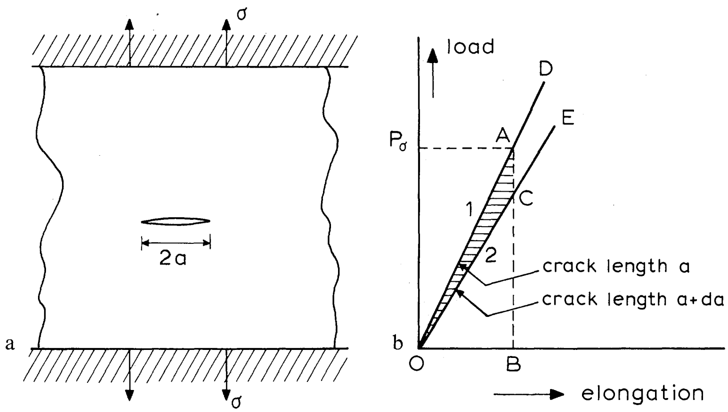

Figure 1.8. The Griffith criterion for fixed grips
a. Cracked plate with fixed ends; b. Elastic energy

and fixed at its ends as in figure 1.8a. The load displacement diagram is given in figure 1.8b. The elastic energy contained in the plate is represented by the area OAB. If the crack extends over a length da the stiffness of the plate will drop (line OC), which means that some load will be relaxed since the ends of the plate are fixed. Consequently, the elastic energy content will drop to a magnitude represented by area OCB. Crack propagation from a to $a+\mathrm{d}a$ will result in an elastic energy release equal in magnitude to area OAC.

If the plate were stressed at a higher stress there would be a larger energy release if the crack grew an amount da. Griffith stated that crack propagation will occur if the energy released upon crack growth is sufficient to provide all the energy that is required for crack growth. If the latter is not the case the stress has to be raised. The triangle ODE represents the amount of energy available if the crack would grow.

The condition for crack growth is:

$$\frac{dU}{da} = \frac{dW}{da} \tag{1.7}$$

where U is the elastic energy and W the energy required for crack growth. Based upon stress field calculations for an elliptical flaw by Inglis [11], Griffith calculated dU/da as:

$$\frac{dU}{da} = \frac{2\pi\sigma^2 a}{E} \tag{1.8}$$

per unit plate thickness, where E is Young's modulus. Usually dU/da is replaced by

$$G = \frac{\pi\sigma^2 a}{E} \tag{1.9}$$

which is the so called "elastic energy release rate" per crack tip. G is also called the crack driving force: its dimensions of energy per unit plate thickness and per unit crack extension are also the dimensions of force per unit crack extension.

The energy consumed in crack propagation is denoted by $R = dW/da$ which is called the crack resistance. To a first approximation it can be assumed that the energy required to produce a crack (the decohesion of atomic bonds) is the same for each increment da. This means that R is a constant.

The energy condition of eq (1.7) now states that G must be at least equal to R before crack propagation can occur. If R is a constant this means that G must exceed a certain critical value G_{Ic}. Hence crack growth occurs when:

$$\frac{\pi\sigma_c^2 a}{E} = G_{Ic} \quad \text{or} \quad \sigma_c = \sqrt{\frac{EG_{Ic}}{\pi a}}. \tag{1.10}$$

The critical value G_{Ic} (critical energy release rate) can be determined by measuring the stress σ_c required to fracture a plate with a crack of size $2a$, and by calculating G_{Ic} from eq (1.10).

Griffith derived his equation for glass, which is a very brittle material. Therefore he assumed that R consisted of surface energy only. In ductile materials, such as metals, plastic deformation occurs at the crack tip. Much work is required in producing a new plastic zone at the tip of the advancing crack. Since this plastic zone has to be produced upon crack growth the energy for its formation can be considered as energy required for crack propagation. This means that for metals R is mainly plastic energy; the surface energy is so small that it can be neglected [12, 13]. The energy criterion is a necessary criterion for crack extension. It need not be a sufficient criterion. Even if sufficient energy for crack propagation can be provided, the crack will not propagate unless the material at the crack tip is ready to fail: the material should be at the end of its capacity to take load and to undergo further straining. However, the latter criterion is equivalent to the energy criterion, since it follows from eqs (1.2) and (1.9) that:

$$\frac{K^2}{E} = G \tag{1.11}$$

Apparently the stress criterion and the energy criterion are fulfilled simultaneously. Hence, eqs (1.4) and (1.10) are equivalent. It is shown in chapter 3 that eq (1.11) is valid for plane stress and that a term $(1-\nu^2)$ has to be added in the case of plane strain, leading to

$$(1 - \nu^2)\frac{K_I^2}{E} = G_I \quad \text{and} \quad (1 - \nu^2)\frac{K_{Ic}^2}{E} = G_{Ic} \tag{1.12}$$

1.5 The crack opening displacement criterion

High strength materials usually have a low fracture toughness. Plane strain fracture problems in these materials can be successfully treated by means of the fracture mechanics procedures described in the two foregoing sections. These procedures are known as the linear elastic fracture mechanics (LEFM) concepts, since they are based on elastic stress field equations. The latter can be used if the size of the crack tip plastic zone is small compared to the size of the crack. According to eq (1.3) the plastic zone size is proportional to K_I^2/σ_{ys}^2. Low strength, low yield strength materials usually have a high toughness. This means that the size of the plastic zone at fracture ($K_I = K_{Ic}$) may be so large as compared to the crack size that LEFM do not apply. The latter is the case if σ_c/σ_{ys} approaches unity. (The size of the plastic zone is also proportional to $(\sigma_c/\sigma_{ys})^2$ as shown in the second eq (1.3).)

At present a versatile method to treat crack problems in high toughness materials is not yet available. Wells [14, 15] has introduced the crack opening displacement (COD) concept for such materials. Supposedly, crack extension can take place when the material at the crack tip has reached a maximum permissible plastic strain. The crack tip strain can be related to the crack opening displacement (chapter 9), which is a measurable quantity.

Crack extension or fracture is assumed to occur as soon as the crack opening displacement exceeds a critical value. It can easily be shown (chapter 9) that this criterion is equivalent to the K_{Ic} and G_{Ic} criterion in the case where LEFM apply. This gives some confidence for the supposed general validity. In the present stage of development, one of the drawbacks of the COD criterion is the fact that it does not permit direct calculation of a fracture stress. The critical COD for high toughness, low strength materials is primarily a comparative toughness parameter.

1.6 Crack propagation

As pointed out in sect. 1.3 the stress intensity factor is a measure for the stress and strain environment of the crack tip. The stress intensity factor is still meaningful if the plastic zone is only small. Then it may also be expected that the rate of fatigue crack propagation per cycle is determined by the stress intensity factor. If two different cracks have the same stress environment, i.e. the same stress intensity factor, they should show the same rate of propagation.

If the fatigue load varies between zero and some positive value (constant amplitude), the stress intensity cycles over a range $\Delta K = K_{max} - K_{min}$, where $K_{min} = 0$. Hence, the rate of fatigue crack propagation per cycle $\mathrm{d}a/\mathrm{d}N$ must depend upon the stress intensity range ΔK:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = f(\Delta K) = f\{2S_a\sqrt{\pi a}\} \tag{1.13}$$

where S_a is the stress amplitude. (The notation S is used for cyclic stresses which is customary in the literature.) Paris, Gomez and Anderson [16] were first to recognize this and to check it with test data. It is obvious that eq (1.13) is satisfied automatically if results of only one test are used: the values of da/dN can always be plotted *versus* the instantaneous values of ΔK, which would show eq (1.13) to be true.

Consider the results of two crack propagation tests in figure 1.9a. The stress amplitudes S_a were the same throughout each test. Apparently, the crack propagation rate progressively increased with increasing crack size.

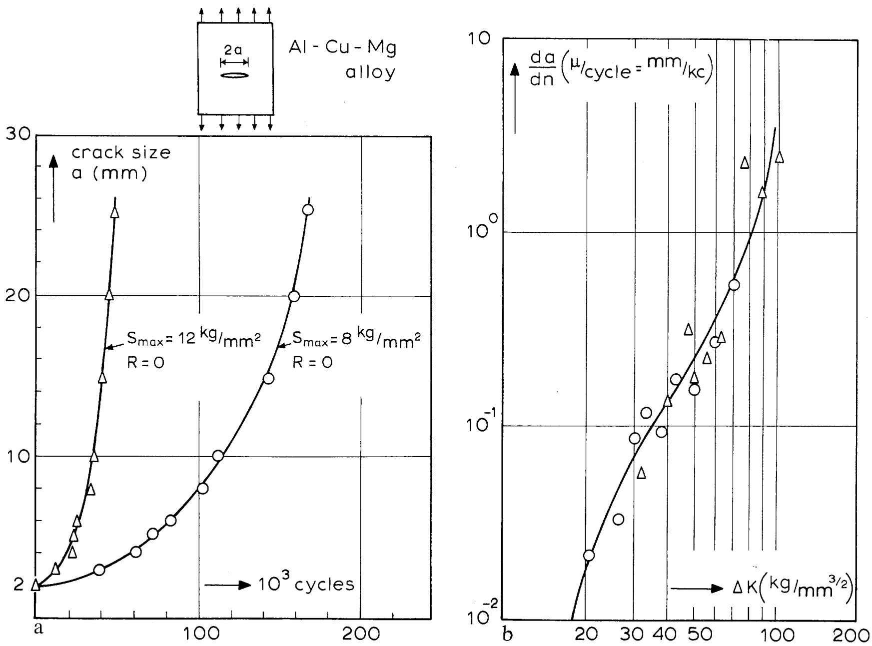

Figure 1.9. Fatigue crack propagation
a. Crack growth curves; b. Crack propagation rate

The rate da/dN can be determined from the slope of the curves. The value of ΔK follows from $\Delta K = 2S_a\sqrt{\pi a}$ by substituting the instantaneous value of a. A double logarithmic plot of da/dN *versus* ΔK is made in figure 1.9b. The data obtained at high stress amplitude start at relatively high values of ΔK and da/dN. The other set of data commences at lower values but reaches the same high values as the other test.

The data of the *two* tests carried out under *different* conditions, are on one single curve, which proves the usefulness of eq (1.13). Apparently it makes no difference whether one has a small crack at high stress or a long crack at low stress: the two will exhibit the same rate of growth if their ΔK is the same.

In a double-logarithmic plot the da/dN *versus* ΔK data often fall on a straight line. Therefore it has been suggested many times that the relation of eq (1.13) should read

$$\frac{da}{dN} = C(\Delta K)^n \tag{1.14}$$

in which C and n are constants. Values of n usually vary between 2 and 4. It turns out, however, that eq (1.14) does not really represent the test data. In practice the plot of da/dN *versus* ΔK is an S-shaped curve, or at least consists of parts with different slopes [17, 18], see figure 1.9. If the tests concern only a limited range of ΔK values the exponential relationship of eq (1.14) is found; but then the value of n depends upon the position of the ΔK range (high, low or intermediate ΔK values). A deviation at the upper end of the ΔK range may be expected when recognizing that the crack is reaching a critical size at which da/dN must become infinite: total failure occurs during the cycle in which the stress intensity reaches K_{Ic}.

A fatigue cycle is determined by two stress parameters, the amplitude S_a and the mean stress S_m. If $S_m = S_a$ the minimum stress in the cycle is zero. That means that the maximum stress intensity in a cycle $K_{max} = \Delta K$. If $S_m > S_a$, the maximum stress intensity $K_{max} = (S_m + S_a)\sqrt{\pi a}$ is larger than ΔK. It is almost self-evident that the maximum stress intensity in a cycle is of influence on the rate of crack growth. Therefore, a more general form of eq (1.13) is:

$$\frac{da}{dN} = f_1(K_{max}, \Delta K) = f_2(R, \Delta K) \text{ with } R = \frac{K_{min}}{K_{max}} = \frac{S_{min}}{S_{max}} = \frac{S_m - S_a}{S_m + S_a}. \tag{1.15}$$

R is known as the cycle ratio (see chapter 10).

Subcritical flaw growth can occur by other mechanisms than fatigue. The most important one is stress corrosion cracking. Given a specific material-environment interaction it is found, as in the case of fatigue crack growth, that the stress corrosion cracking rate (and hence the time to failure) is governed by the stress intensity factor. Similar specimens with the same initial crack but loaded at different levels (different initial

K-values) show different times to failure [19] as is shown diagrammatically in figure 1.10. The specimen initially loaded to K_{Ic} fails immediately. Specimens subjected to K values below a certain threshold level never fail. This threshold level is denoted as K_{Iscc}, scc standing for stress corrosion cracking.

During the stress corrosion cracking process the load can be kept constant. Since the crack extends, the stress intensity gradually increases. As a result the crack growth rate per unit of time, $\mathrm{d}a/\mathrm{d}t$, increases according to:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = f(K)\,. \tag{1.16}$$

When the crack has grown to a size such that K becomes equal to K_{Ic}, final failure occurs, as indicated in figure 1.11.

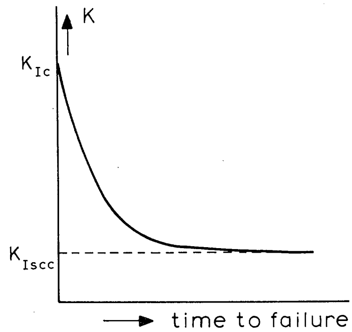

Figure 1.10. Stress corrosion time to failure, upon loading to initial K-level

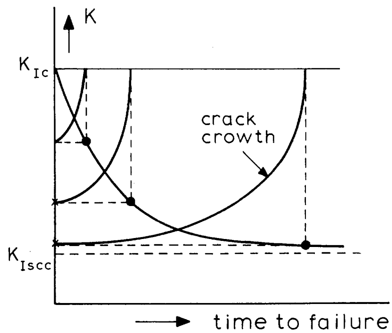

Figure 1.11. Stress corrosion cracking

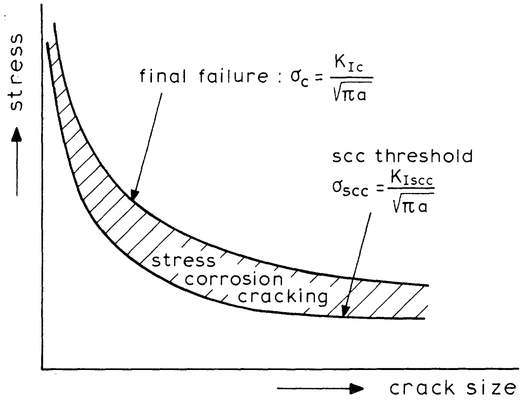

Figure 1.12. Relation between stress and crack length for stress corrosion to occur

The stress corrosion cracking threshold, K_{Iscc}, and the rate of crack growth depends upon the material and the environmental conditions. From figure 1.12 it follows that a component with a certain size of crack loaded to a stress σ, such that $\sigma\sqrt{\pi a} = K_{Ic}$, fails on initial loading. Components loaded to K values at or above K_{Iscc} (shaded area) will show crack growth to failure. Although fracture mechanics concepts can be applied to stress corrosion cracking, the predictive power of these concepts is still very limited. Therefore the problem of stress corrosion cracking receives only limited attention in this volume.

1.7 Closure

It was shown that the stress intensity factor describes the stress field and the plastic zone. Equal stress intensity factors will give equal stress fields and equal plastic zones. This similitude at the crack tip demands that equal things happen so that equal stress-intensity factors should cause equal growth rates. Fracture occurs when K exceeds a critical value, K_{Ic}.

The similitude concept is used quite extensively in engineering: e.g. yielding occurs when the stress, σ, exceeds a critical value, σ_{ys}. A remaining question is whether equal K always guarantees crack tip similitude. As will appear in following chapters additional similitude requirements are often necessary.

In principle, knowledge of the stress intensity factor for a crack in a particular structural element enables prediction of crack growth and fracture. Unfortunately, so many complications occur – often because of additional similitude requirements – that it is not always possible to apply the simple concepts outlined in this chapter. However, useful solutions can be obtained in many cases, but they involve engineering judgement. The latter has to be based on a fair knowledge of the physical principles and assumptions which will be dealt with in the subsequent chapters.

References

[1] Anderson, W. E., An engineer views brittle fracture history, *Boeing rept.*, (1969).
[2] Biggs, W. D., *The brittle fracture of steel*, McDonald and Evans (1960).
[3] Muskhelishvili, N. I., *Some basic problems of mathematical theory of elasticity*, (1933). English translation, Noordhoff (1953).
[4] Westergaard, H. M., Bearing pressures and cracks, *J. Appl. Mech.*, 61 (1939) pp. A49–A53.
[5] Paris, P. C. and Sih, G. C., Stress analysis of cracks, *ASTM STP* 381, (1965) pp. 30–81.
[6] Eshelby, J. D., Stress analysis: elasticity and fracture mechanics, *ISI publ. 121*, (1968) pp. 13–48.
[7] Irwin, G. R., Fracture I, *Handbuch der Physik VI*, Flügge Ed., pp. 558–590, Springer, (1958).
[8] McClintock, F. A. and Irwin, G. R., Plasticity aspects of fracture mechanics, *ASTM STP 381*, (1965) pp. 84–113.
[9] Griffith, A. A., The phenomena of rupture and flow in solids, *Phil. Trans. Roy. Soc. of London*, A 221 (1921) pp. 163–197.
[10] Griffith, A. A., The theory of rupture, *Proc. 1st Int. Congress Appl. Mech.*, (1924) pp. 55–63. Biezeno and Burgers ed. Waltman, (1925).
[11] Inglis, C. E., Stresses in a plate due to the presence of cracks and sharp corners, *Trans. Inst. Naval Architects*, 55 (1913) pp. 219–241.
[12] Orowan, E., Energy criteria of fracture, *Welding Journal*, 34 (1955) pp. 1575–1605.
[13] Irwin, G. R., Fracture dynamics, *Fracturing of Metals*, pp. 147–166, ASM publ. (1948).
[14] Wells, A. A., *Unstable crack propagation in metals, cleavage and fast fracture*, The Crack Propagation Symposium, pp. 210–230, Cranfield, (1961).
[15] Wells, A. A., Application of fracture mechanics at and beyond general yield, *British Welding Res. Ass. Rept.*, M13/63 (1963).
[16] Paris, P. C., Gomez, M. P. and Anderson, W. E., A rational analytic theory of fatigue, *The Trend in Engineering*, 13 (1961) pp. 9–14.
[17] Broek, D. and Schijve, J., The influence of the mean stress on fatigue crack propagation, *Aircraft Engineering*, 39 (1967) pp. 10–13.
[18] Wilhem, D. P., Investigation of cyclic crack growth transitional behaviour, *ASTM STP 415*, (1967) pp. 363–383.
[19] Brown, B. F., The application of fracture mechanics to scc, *Metals and Materials*, 2 (1968); *Met. Reviews*, 13 (1968) pp. 171–183.

Unité du Facteur d'intensité de contrainte

En applications géologique actuelles, il est donné en $\text{MPa}\sqrt{\text{m}}$

1 Pa = 1 N/m^2

On a donc $\dfrac{\text{MN}}{m^2}\sqrt{\text{m}} = \text{MN}\,m^{-2}\sqrt{\text{m}}$

$a^p\ a^q = a^{p+q}$

$\text{m}^{-2+\frac{1}{2}} = \text{m}^{\frac{-4+1}{2}} = \text{m}^{-\frac{3}{2}}$

$a^{-n/m} = 1/a^{n/m}$

et Broek donne des valeurs, par exemple en $MN/m^{3/2}$